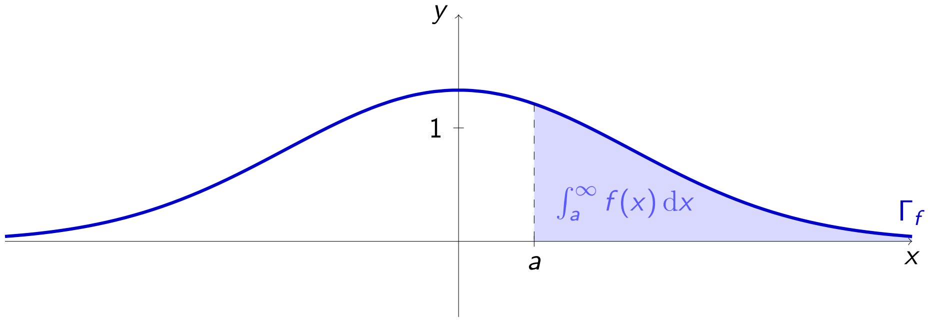
5.7.1. Nepravi integrali s neograničenim područjem integriranja

8. 1. 2020.

Fotografija: Kalen Emsley

Definicija 1(a)

Neka su $a \in \mathbb{R}$ i $f : D \subseteq \mathbb{R} \to \mathbb{R}$, pri čemu $D \supseteq [a, +\infty\rangle$.



Definiramo

$$\int_a^\infty f(x)\,\mathrm{d}x :=$$

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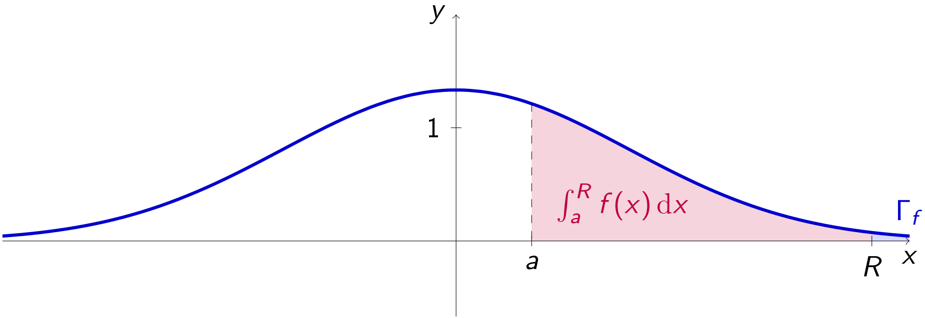

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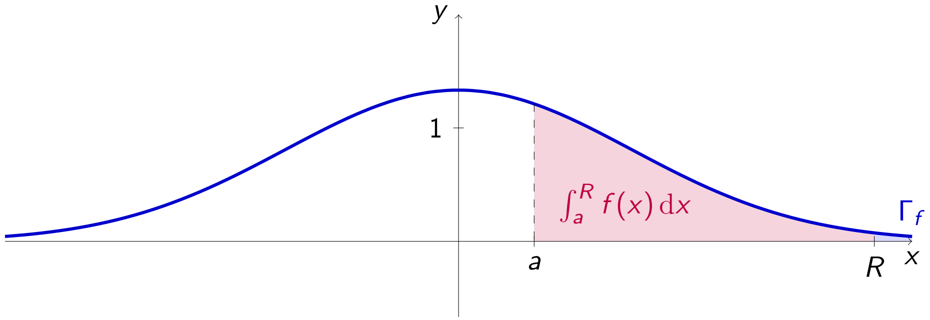


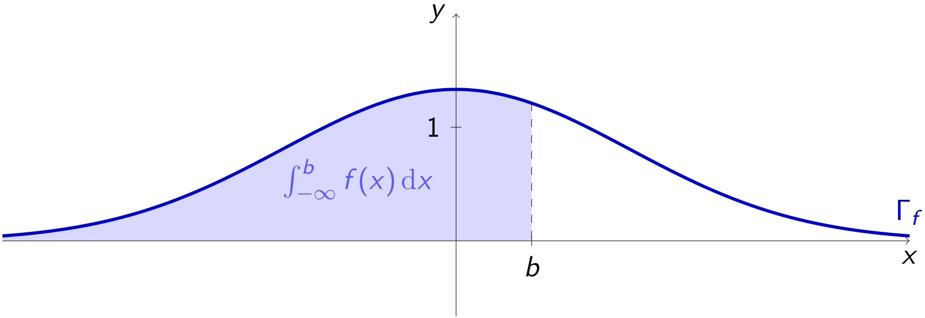
Definiramo

$$\int_a^{\infty} f(x)\,\mathrm{d}x := \lim_{R \to +\infty} \int_a^R f(x)\,\mathrm{d}x,$$

kad god je desna strana ove formule definirana.

Definicija 1(b)

Neka su $b \in \mathbb{R}$ i $f : D \subseteq \mathbb{R} \to \mathbb{R}$, pri čemu $D \supseteq \langle -\infty, b]$.



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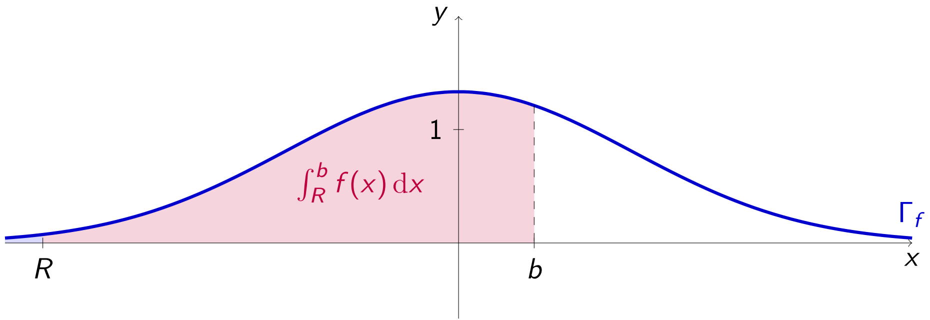

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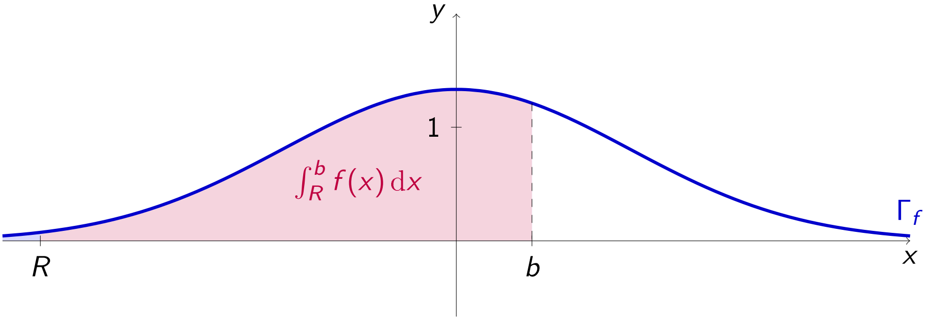


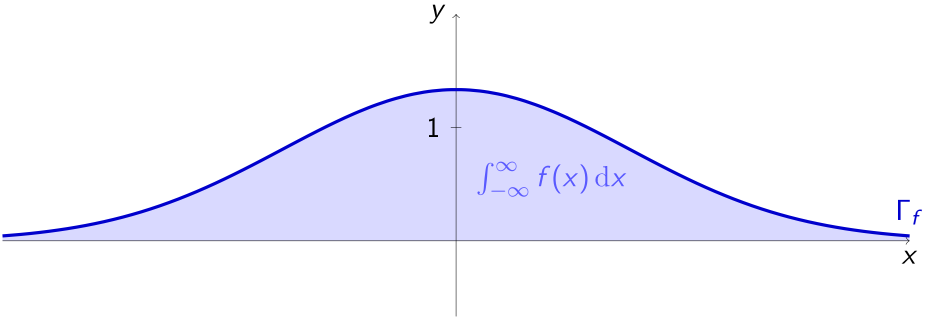
Definiramo

$$\int_{-\infty}^{b} f(x)\,\mathrm{d}x := \lim_{R \to -\infty} \int_{R}^{b} f(x)\,\mathrm{d}x,$$

kad god je desna strana ove formule definirana.

Definicija 1(c)

Neka je $f : \mathbb{R} \to \mathbb{R}$.



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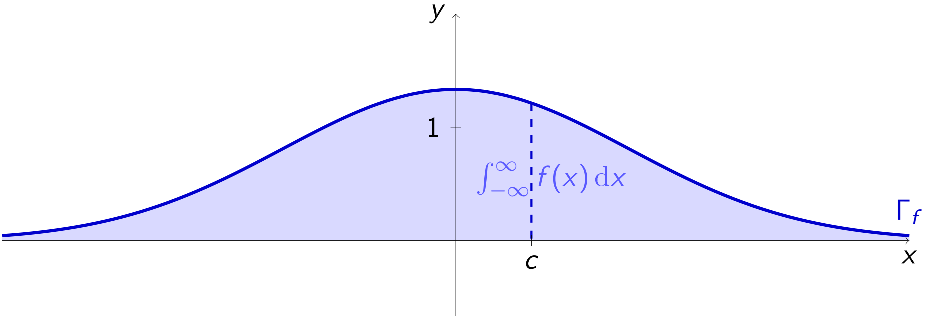

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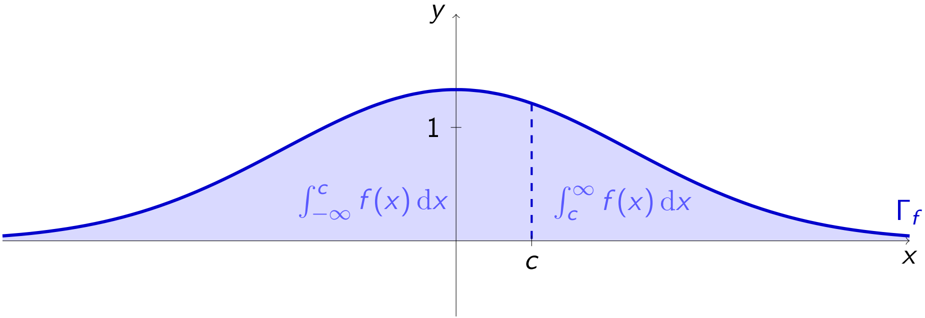

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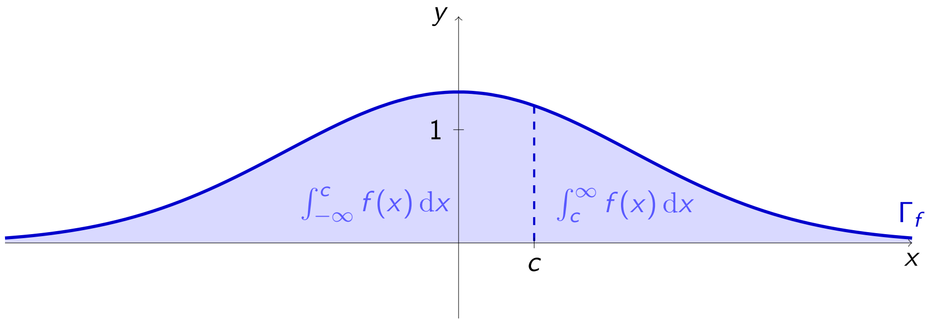

Definiramo

$$\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x := \int_{-\infty}^{c} f(x)\,\mathrm{d}x + \int_{c}^{\infty} f(x)\,\mathrm{d}x$$

za bilo koji $c \in \mathbb{R}$, kad god je desna strana ove formule definirana. Ova definicija ne ovisi o izboru $c \in \mathbb{R}$.

Zadatak 53(a)

Izračunajte integral $\int_0^{\infty} \frac{\mathrm{d}x}{1+x^2}$.

$$\int \mathrm{d}x = x + C$$
$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
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$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \; a \neq 1)$$
$$\int \cos x \, \mathrm{d}x = \sin x + C$$
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Rješenje. Koristeći definiciju nepravog integrala

$$\int_a^{\infty} f(x)\,\mathrm{d}x := \lim_{R\to+\infty} \int_a^R f(x)\,\mathrm{d}x,$$

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$$\int_0^\infty \frac{\mathrm{d}x}{1+x^2} = \lim_{R\to+\infty}\int_0^R \frac{\mathrm{d}x}{1+x^2}$$

$$= \lim_{R\to+\infty} \operatorname{arctg} x\Big|_0^R$$

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$$\begin{aligned}\int_0^{\infty} \frac{\mathrm{d}x}{1+x^2} &= \lim_{R\to+\infty} \int_0^R \frac{\mathrm{d}x}{1+x^2} \\ &= \lim_{R\to+\infty} \operatorname{arctg} x \Big|_0^R \\ &= \lim_{R\to+\infty} (\operatorname{arctg} R - \operatorname{arctg} 0)\end{aligned}$$

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&= \lim_{R\to+\infty} (\operatorname{arctg} R - \operatorname{arctg} 0) \\
&= \lim_{R\to+\infty} \operatorname{arctg} R \\
&= \frac{\pi}{2}.
\end{aligned}$$

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Zadatak 53(b)

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$$= \lim_{R\to-\infty} \left(-\frac{1}{x}\right)\Big|_{R}^{a}$$

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$$= \lim_{R\to-\infty} \left(-\frac{1}{x}\right)\Bigg|_{R}^{a}$$

$$= \lim_{R\to-\infty} \left(-\frac{1}{a} + \frac{1}{R}\right)$$

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$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(b)

Neka je $a \in \langle -\infty, 0\rangle$. Izračunajte integral $\displaystyle\int_{-\infty}^{a} \frac{\mathrm{d}x}{x^2}$.

Rješenje. Koristeći definiciju nepravog integrala

$$\int_{-\infty}^{b} f(x)\,\mathrm{d}x := \lim_{R\to-\infty} \int_{R}^{b} f(x)\,\mathrm{d}x,$$

imamo

$$\begin{aligned}
\int_{-\infty}^{a} \frac{\mathrm{d}x}{x^2} &= \lim_{R\to-\infty} \int_{R}^{a} \frac{\mathrm{d}x}{x^2} \\
&= \lim_{R\to-\infty} \left(-\frac{1}{x}\right)\Bigg|_{R}^{a} \\
&= \lim_{R\to-\infty} \left(-\frac{1}{a} + \frac{1}{R}\right) \\
&= -\frac{1}{a} + 0
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(b)

Neka je $a \in \langle -\infty, 0\rangle$. Izračunajte integral $\displaystyle\int_{-\infty}^{a} \frac{\mathrm{d}x}{x^2}$.

Rješenje. Koristeći definiciju nepravog integrala

$$\int_{-\infty}^{b} f(x)\,\mathrm{d}x := \lim_{R\to-\infty} \int_{R}^{b} f(x)\,\mathrm{d}x,$$

imamo

$$\begin{aligned}
\int_{-\infty}^{a} \frac{\mathrm{d}x}{x^2} &= \lim_{R\to-\infty} \int_{R}^{a} \frac{\mathrm{d}x}{x^2} \\
&= \lim_{R\to-\infty} \left(-\frac{1}{x}\right)\Big|_{R}^{a} \\
&= \lim_{R\to-\infty} \left(-\frac{1}{a} + \frac{1}{R}\right) \\
&= -\frac{1}{a} + 0 = -\frac{1}{a}.
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(c)

Izračunajte integral $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2 + 6x + 10}$.

Zadatak 53(c)

Izračunajte integral $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2 + 6x + 10}$.

Rješenje. Koristeći definiciju nepravog integrala

$$\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x := \int_{-\infty}^{c} f(x)\,\mathrm{d}x + \int_{c}^{\infty} f(x)\,\mathrm{d}x$$

za bilo koji $c \in \mathbb{R}$,

Zadatak 53(c)

Izračunajte integral $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2 + 6x + 10}$.

Rješenje. Koristeći definiciju nepravog integrala

$$\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x := \int_{-\infty}^{c} f(x)\,\mathrm{d}x + \int_{c}^{\infty} f(x)\,\mathrm{d}x$$

za bilo koji $c \in \mathbb{R}$, imamo, stavljajući (npr.) $c = 0$,

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2 + 6x + 10} = \int_{-\infty}^{0} \frac{\mathrm{d}x}{x^2 + 6x + 10} + \int_{0}^{\infty} \frac{\mathrm{d}x}{x^2 + 6x + 10}$$

Zadatak 53(c)

Izračunajte integral $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10}$.

Rješenje. Koristeći definiciju nepravog integrala

$$\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x := \int_{-\infty}^{c} f(x)\,\mathrm{d}x + \int_{c}^{\infty} f(x)\,\mathrm{d}x$$

za bilo koji $c \in \mathbb{R}$, imamo, stavljajući (npr.) $c = 0$,

$$\begin{aligned}\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10} &= \int_{-\infty}^{0} \frac{\mathrm{d}x}{x^2+6x+10} + \int_{0}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10}\\ &= \lim_{R\to-\infty} \int_{R}^{0} \frac{\mathrm{d}x}{x^2+6x+10} + \lim_{R\to+\infty} \int_{0}^{R} \frac{\mathrm{d}x}{x^2+6x+10}.\end{aligned}$$

Zadatak 53(c)

Izračunajte integral $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10}$.

Rješenje (nastavak). Dakle,

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10} = \lim_{R\to-\infty} \int_{R}^{0} \frac{\mathrm{d}x}{x^2+6x+10} + \lim_{R\to+\infty} \int_{0}^{R} \frac{\mathrm{d}x}{x^2+6x+10}$$

Zadatak 53(c)

Izračunajte integral $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10}$.

Rješenje (nastavak). Dakle,

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10} = \lim_{R\to-\infty} \int_{R}^{0} \frac{\mathrm{d}x}{x^2+6x+10} + \lim_{R\to+\infty} \int_{0}^{R} \frac{\mathrm{d}x}{x^2+6x+10}$$

$$\int \frac{\mathrm{d}x}{x^2+6x+10} =$$

Zadatak 53(c)

Izračunajte integral $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10}$.

Rješenje (nastavak). Dakle,

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10} = \lim_{R\to-\infty} \int_R^0 \frac{\mathrm{d}x}{x^2+6x+10} + \lim_{R\to+\infty} \int_0^R \frac{\mathrm{d}x}{x^2+6x+10}$$

$$\int \frac{\mathrm{d}x}{x^2+6x+10} = \int \frac{\mathrm{d}x}{(x+3)^2+1}$$

Zadatak 53(c)

Izračunajte integral $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10}$.

Rješenje (nastavak). Dakle,

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10} = \lim_{R\to-\infty} \int_R^0 \frac{\mathrm{d}x}{x^2+6x+10} + \lim_{R\to+\infty} \int_0^R \frac{\mathrm{d}x}{x^2+6x+10}$$

$$\int \frac{\mathrm{d}x}{x^2+6x+10} = \int \frac{\mathrm{d}x}{(x+3)^2+1} = \begin{bmatrix} t = x+3 \\ \mathrm{d}t = \mathrm{d}x \end{bmatrix}$$

Zadatak 53(c)

Izračunajte integral $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2 + 6x + 10}$.

Rješenje (nastavak). Dakle,

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2 + 6x + 10} = \lim_{R \to -\infty} \int_{R}^{0} \frac{\mathrm{d}x}{x^2 + 6x + 10} + \lim_{R \to +\infty} \int_{0}^{R} \frac{\mathrm{d}x}{x^2 + 6x + 10}$$

$$\int \frac{\mathrm{d}x}{x^2 + 6x + 10} = \int \frac{\mathrm{d}x}{(x+3)^2 + 1} = \begin{bmatrix} t = x + 3 \\ \mathrm{d}t = \mathrm{d}x \end{bmatrix} = \int \frac{\mathrm{d}t}{t^2 + 1}$$

Zadatak 53(c)

Izračunajte integral $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10}$.

Rješenje (nastavak). Dakle,

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10} = \lim_{R\to-\infty} \int_{R}^{0} \frac{\mathrm{d}x}{x^2+6x+10} + \lim_{R\to+\infty} \int_{0}^{R} \frac{\mathrm{d}x}{x^2+6x+10}$$

$$\int \frac{\mathrm{d}x}{x^2+6x+10} = \int \frac{\mathrm{d}x}{(x+3)^2+1} = \begin{bmatrix} t = x+3 \\ \mathrm{d}t = \mathrm{d}x \end{bmatrix} = \int \frac{\mathrm{d}t}{t^2+1} = \operatorname{arctg} t + C$$

Zadatak 53(c)

Izračunajte integral $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10}$.

Rješenje (nastavak). Dakle,

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10} = \lim_{R\to-\infty} \int_{R}^{0} \frac{\mathrm{d}x}{x^2+6x+10} + \lim_{R\to+\infty} \int_{0}^{R} \frac{\mathrm{d}x}{x^2+6x+10}$$

$$\int \frac{\mathrm{d}x}{x^2+6x+10} = \int \frac{\mathrm{d}x}{(x+3)^2+1} = \begin{bmatrix} t = x+3 \\ \mathrm{d}t = \mathrm{d}x \end{bmatrix} = \int \frac{\mathrm{d}t}{t^2+1} = \operatorname{arctg} t + C = \operatorname{arctg}(x+3) + C.$$

Zadatak 53(c)

Izračunajte integral $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10}$.

Rješenje (nastavak). Dakle,

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10} = \lim_{R\to-\infty} \int_{R}^{0} \frac{\mathrm{d}x}{x^2+6x+10} + \lim_{R\to+\infty} \int_{0}^{R} \frac{\mathrm{d}x}{x^2+6x+10}$$

$$= \lim_{R\to-\infty} \operatorname{arctg}(x+3)\Big|_{R}^{0} + \lim_{R\to+\infty} \operatorname{arctg}(x+3)\Big|_{0}^{R}$$

$$\int \frac{\mathrm{d}x}{x^2+6x+10} = \int \frac{\mathrm{d}x}{(x+3)^2+1} = \begin{bmatrix} t = x+3 \\ \mathrm{d}t = \mathrm{d}x \end{bmatrix} = \int \frac{\mathrm{d}t}{t^2+1} = \operatorname{arctg} t + C = \operatorname{arctg}(x+3) + C.$$

Zadatak 53(c)

Izračunajte integral $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10}$.

Rješenje (nastavak). Dakle,

$$\begin{aligned}
\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10} &= \lim_{R\to-\infty} \int_{R}^{0} \frac{\mathrm{d}x}{x^2+6x+10} + \lim_{R\to+\infty} \int_{0}^{R} \frac{\mathrm{d}x}{x^2+6x+10} \\
&= \lim_{R\to-\infty} \operatorname{arctg}(x+3)\Big|_{R}^{0} + \lim_{R\to+\infty} \operatorname{arctg}(x+3)\Big|_{0}^{R} \\
&= \lim_{R\to-\infty} (\operatorname{arctg} 3 - \operatorname{arctg}(R+3)) + \lim_{R\to+\infty} (\operatorname{arctg}(R+3) - \operatorname{arctg} 3)
\end{aligned}$$

$$\int \frac{\mathrm{d}x}{x^2+6x+10} = \int \frac{\mathrm{d}x}{(x+3)^2+1} = \begin{bmatrix} t = x+3 \\ \mathrm{d}t = \mathrm{d}x \end{bmatrix} = \int \frac{\mathrm{d}t}{t^2+1} = \operatorname{arctg} t + C = \operatorname{arctg}(x+3) + C.$$

Zadatak 53(c)

Izračunajte integral $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10}$.

Rješenje (nastavak). Dakle,

$$\begin{aligned}
\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10} &= \lim_{R\to-\infty} \int_{R}^{0} \frac{\mathrm{d}x}{x^2+6x+10} + \lim_{R\to+\infty} \int_{0}^{R} \frac{\mathrm{d}x}{x^2+6x+10} \\
&= \lim_{R\to-\infty} \operatorname{arctg}(x+3)\Big|_{R}^{0} + \lim_{R\to+\infty} \operatorname{arctg}(x+3)\Big|_{0}^{R} \\
&= \lim_{R\to-\infty} (\operatorname{arctg} 3 - \operatorname{arctg}(R+3)) + \lim_{R\to+\infty} (\operatorname{arctg}(R+3) - \operatorname{arctg} 3) \\
&= \operatorname{arctg} 3 - \left(-\frac{\pi}{2}\right) + \frac{\pi}{2} - \operatorname{arctg} 3
\end{aligned}$$

$$\int \frac{\mathrm{d}x}{x^2+6x+10} = \int \frac{\mathrm{d}x}{(x+3)^2+1} = \begin{bmatrix} t = x+3 \\ \mathrm{d}t = \mathrm{d}x \end{bmatrix} = \int \frac{\mathrm{d}t}{t^2+1} = \operatorname{arctg} t + C = \operatorname{arctg}(x+3) + C.$$

Zadatak 53(c)

Izračunajte integral $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10}$.

Rješenje (nastavak). Dakle,

$$\begin{aligned}
\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10} &= \lim_{R\to-\infty}\int_R^0 \frac{\mathrm{d}x}{x^2+6x+10} + \lim_{R\to+\infty}\int_0^R \frac{\mathrm{d}x}{x^2+6x+10} \\
&= \lim_{R\to-\infty} \operatorname{arctg}(x+3)\Big|_R^0 + \lim_{R\to+\infty} \operatorname{arctg}(x+3)\Big|_0^R \\
&= \lim_{R\to-\infty} (\operatorname{arctg} 3 - \operatorname{arctg}(R+3)) + \lim_{R\to+\infty} (\operatorname{arctg}(R+3) - \operatorname{arctg} 3) \\
&= \cancel{\operatorname{arctg} 3} - \left(-\frac{\pi}{2}\right) + \frac{\pi}{2} - \cancel{\operatorname{arctg} 3}
\end{aligned}$$

$$\int \frac{\mathrm{d}x}{x^2+6x+10} = \int \frac{\mathrm{d}x}{(x+3)^2+1} = \begin{bmatrix} t = x+3 \\ \mathrm{d}t = \mathrm{d}x \end{bmatrix} = \int \frac{\mathrm{d}t}{t^2+1} = \operatorname{arctg} t + C = \operatorname{arctg}(x+3) + C.$$

Zadatak 53(c)

Izračunajte integral $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10}$.

Rješenje (nastavak). Dakle,

$$\begin{aligned}
\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^2+6x+10} &= \lim_{R\to-\infty} \int_{R}^{0} \frac{\mathrm{d}x}{x^2+6x+10} + \lim_{R\to+\infty} \int_{0}^{R} \frac{\mathrm{d}x}{x^2+6x+10} \\
&= \lim_{R\to-\infty} \operatorname{arctg}(x+3)\Big|_{R}^{0} + \lim_{R\to+\infty} \operatorname{arctg}(x+3)\Big|_{0}^{R} \\
&= \lim_{R\to-\infty} (\operatorname{arctg} 3 - \operatorname{arctg}(R+3)) + \lim_{R\to+\infty} (\operatorname{arctg}(R+3) - \operatorname{arctg} 3) \\
&= \cancel{\operatorname{arctg} 3} - \left(-\frac{\pi}{2}\right) + \frac{\pi}{2} - \cancel{\operatorname{arctg} 3} = \pi,
\end{aligned}$$

pri čemu druga jednakost vrijedi jer je

$$\int \frac{\mathrm{d}x}{x^2+6x+10} = \int \frac{\mathrm{d}x}{(x+3)^2+1} = \begin{bmatrix} t = x+3 \\ \mathrm{d}t = \mathrm{d}x \end{bmatrix} = \int \frac{\mathrm{d}t}{t^2+1} = \operatorname{arctg} t + C = \operatorname{arctg}(x+3) + C.$$

Zadatak 53(d)

Izračunajte integral $\int_e^{\infty} \frac{dx}{x \ln^3 x}$.

$$\int dx = x + C$$

$$\int x^a \, dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int e^x \, dx = e^x + C$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C \quad (a > 0, \ a \neq 1)$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{dx}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(d)

Izračunajte integral $\displaystyle\int_e^{\infty} \frac{\mathrm{d}x}{x\ln^3 x}$.

Rješenje. Imamo

$$\int_e^{\infty} \frac{\mathrm{d}x}{x\ln^3 x} = \lim_{R\to+\infty} \int_e^{R} \frac{\mathrm{d}x}{x\ln^3 x}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(d)

Izračunajte integral $\int_e^{\infty} \frac{\mathrm{d}x}{x \ln^3 x}$.

Rješenje. Imamo

$$\int_e^{\infty} \frac{\mathrm{d}x}{x \ln^3 x} = \lim_{R \to +\infty} \int_e^{R} \frac{\mathrm{d}x}{x \ln^3 x}$$

$$= \begin{bmatrix} t = \ln x & e \mapsto 1 \\ \mathrm{d}t = \frac{\mathrm{d}x}{x} & R \mapsto \ln R \end{bmatrix}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \,\mathrm{d}x = e^x + C$$

$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x \,\mathrm{d}x = \sin x + C$$

$$\int \sin x \,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1 - x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1 + x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(d)

Izračunajte integral $\displaystyle\int_e^{\infty} \frac{\mathrm{d}x}{x \ln^3 x}$.

Rješenje. Imamo

$$\int_e^{\infty} \frac{\mathrm{d}x}{x \ln^3 x} = \lim_{R\to+\infty} \int_e^{R} \frac{\mathrm{d}x}{x \ln^3 x}$$

$$= \begin{bmatrix} t = \ln x & e \mapsto 1 \\ \mathrm{d}t = \frac{\mathrm{d}x}{x} & R \mapsto \ln R \end{bmatrix} = \lim_{R\to+\infty} \int_1^{\ln R} \frac{\mathrm{d}t}{t^3}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \,\mathrm{d}x = e^x + C$$

$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x \,\mathrm{d}x = \sin x + C$$

$$\int \sin x \,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(d)

Izračunajte integral $\int_e^{\infty} \frac{\mathrm{d}x}{x \ln^3 x}$.

Rješenje. Imamo

$$\int_e^{\infty} \frac{\mathrm{d}x}{x \ln^3 x} = \lim_{R\to+\infty} \int_e^{R} \frac{\mathrm{d}x}{x \ln^3 x}$$

$$= \begin{bmatrix} t = \ln x & e \mapsto 1 \\ \mathrm{d}t = \frac{\mathrm{d}x}{x} & R \mapsto \ln R \end{bmatrix} = \lim_{R\to+\infty} \int_1^{\ln R} \frac{\mathrm{d}t}{t^3}$$

$$= \lim_{R\to+\infty} \left(-\frac{1}{2t^2}\right) \Big|_1^{\ln R}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \,\mathrm{d}x = e^x + C$$

$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x \,\mathrm{d}x = \sin x + C$$

$$\int \sin x \,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(d)

Izračunajte integral $\displaystyle\int_e^{\infty} \frac{\mathrm{d}x}{x\ln^3 x}$.

Rješenje. Imamo

$$\begin{aligned}
\int_e^{\infty} \frac{\mathrm{d}x}{x\ln^3 x} &= \lim_{R\to+\infty}\int_e^{R} \frac{\mathrm{d}x}{x\ln^3 x} \\
&= \begin{bmatrix} t=\ln x & e\mapsto 1 \\ \mathrm{d}t=\frac{\mathrm{d}x}{x} & R\mapsto \ln R \end{bmatrix} = \lim_{R\to+\infty}\int_1^{\ln R} \frac{\mathrm{d}t}{t^3} \\
&= \lim_{R\to+\infty}\left(-\frac{1}{2t^2}\right)\Big|_1^{\ln R} \\
&= \lim_{R\to+\infty}\left(-\frac{1}{2\ln^2 R}+\frac{1}{2}\right)
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a\neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a>0,\ a\neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a>0)$$

Zadatak 53(d)

Izračunajte integral $\int_e^{\infty} \frac{\mathrm{d}x}{x \ln^3 x}$.

Rješenje. Imamo

$$\begin{aligned}
\int_e^{\infty} \frac{\mathrm{d}x}{x \ln^3 x} &= \lim_{R\to+\infty} \int_e^{R} \frac{\mathrm{d}x}{x \ln^3 x} \\
&= \begin{bmatrix} t = \ln x & e \mapsto 1 \\ \mathrm{d}t = \frac{\mathrm{d}x}{x} & R \mapsto \ln R \end{bmatrix} = \lim_{R\to+\infty} \int_1^{\ln R} \frac{\mathrm{d}t}{t^3} \\
&= \lim_{R\to+\infty} \left(-\frac{1}{2t^2} \right) \Bigg|_1^{\ln R} \\
&= \lim_{R\to+\infty} \left(-\frac{1}{2 \ln^2 R} + \frac{1}{2} \right) \\
&= \frac{1}{2}.
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x \,\mathrm{d}x = e^x + C$$
$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x \,\mathrm{d}x = \sin x + C$$
$$\int \sin x \,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(e)

Izračunajte integral $\int_1^{\infty} \frac{x^2+1}{x^3}\,\mathrm{d}x$.

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(e)

Izračunajte integral $\int_1^{\infty} \frac{x^2+1}{x^3}\,\mathrm{d}x$.

Rješenje. Imamo

$$\int_1^{\infty} \frac{x^2+1}{x^3}\,\mathrm{d}x = \lim_{R\to+\infty} \int_1^{R} \left(\frac{1}{x} + \frac{1}{x^3}\right)\,\mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(e)

Izračunajte integral $\int_1^{\infty} \frac{x^2+1}{x^3}\,\mathrm{d}x$.

Rješenje. Imamo

$$\begin{aligned}\int_1^{\infty} \frac{x^2+1}{x^3}\,\mathrm{d}x &= \lim_{R\to+\infty} \int_1^{R} \left(\frac{1}{x} + \frac{1}{x^3}\right)\,\mathrm{d}x \\ &= \lim_{R\to+\infty} \left(\ln|x| - \frac{1}{2x^2}\right)\Bigg|_1^R\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(e)

Izračunajte integral $\int_1^{\infty} \frac{x^2+1}{x^3}\,\mathrm{d}x$.

Rješenje. Imamo

$$\begin{aligned}\int_1^{\infty} \frac{x^2+1}{x^3}\,\mathrm{d}x &= \lim_{R\to+\infty} \int_1^{R} \left(\frac{1}{x} + \frac{1}{x^3}\right)\,\mathrm{d}x \\ &= \lim_{R\to+\infty} \left(\ln|x| - \frac{1}{2x^2}\right)\Bigg|_1^R \\ &= \lim_{R\to+\infty} \left(\ln R - \frac{1}{2R^2} - \left(\ln 1 - \frac{1}{2}\right)\right)\end{aligned}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(e)

Izračunajte integral $\int_1^{\infty} \frac{x^2+1}{x^3}\,\mathrm{d}x.$

Rješenje. Imamo

$$\begin{aligned}
\int_1^{\infty} \frac{x^2+1}{x^3}\,\mathrm{d}x &= \lim_{R\to+\infty} \int_1^{R} \left(\frac{1}{x} + \frac{1}{x^3}\right)\,\mathrm{d}x \\
&= \lim_{R\to+\infty} \left(\ln|x| - \frac{1}{2x^2}\right)\Bigg|_1^R \\
&= \lim_{R\to+\infty} \left(\ln R - \frac{1}{2R^2} - \left(\ln 1 - \frac{1}{2}\right)\right) \\
&= \left((+\infty) - 0 - \left(-\frac{1}{2}\right)\right)
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(e)

Izračunajte integral $\displaystyle\int_1^{\infty} \frac{x^2+1}{x^3}\,\mathrm{d}x.$

Rješenje. Imamo

$$
\begin{aligned}
\int_1^{\infty} \frac{x^2+1}{x^3}\,\mathrm{d}x &= \lim_{R\to+\infty} \int_1^{R} \left(\frac{1}{x} + \frac{1}{x^3}\right)\,\mathrm{d}x \\
&= \lim_{R\to+\infty} \left(\ln|x| - \frac{1}{2x^2}\right)\Bigg|_1^R \\
&= \lim_{R\to+\infty} \left(\ln R - \frac{1}{2R^2} - \left(\ln 1 - \frac{1}{2}\right)\right) \\
&= \left((+\infty) - 0 - \left(-\frac{1}{2}\right)\right) \\
&= +\infty.
\end{aligned}
$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(f)

Izračunajte integral $\int_0^{\infty} \sin x \, \mathrm{d}x$.

$$\int \mathrm{d}x = x + C$$
$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x \, \mathrm{d}x = e^x + C$$
$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \ a \neq 1)$$
$$\int \cos x \, \mathrm{d}x = \sin x + C$$
$$\int \sin x \, \mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(f)

Izračunajte integral $\int_0^{\infty} \sin x \, \mathrm{d}x$.

Rješenje. Imamo

$$\int_0^{\infty} \sin x \, \mathrm{d}x = \lim_{R \to +\infty} \int_0^R \sin x \, \mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \ a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1 - x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1 + x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(f)

Izračunajte integral $\displaystyle\int_0^{\infty} \sin x \,\mathrm{d}x$.

Rješenje. Imamo

$$\int_0^{\infty} \sin x \,\mathrm{d}x = \lim_{R\to+\infty} \int_0^{R} \sin x \,\mathrm{d}x$$

$$= \lim_{R\to+\infty} (-\cos x)\Big|_0^R$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \,\mathrm{d}x = e^x + C$$

$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x \,\mathrm{d}x = \sin x + C$$

$$\int \sin x \,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(f)

Izračunajte integral $\int_0^{\infty} \sin x \, \mathrm{d}x$.

Rješenje. Imamo

$$\begin{aligned}\int_0^{\infty} \sin x \, \mathrm{d}x &= \lim_{R \to +\infty} \int_0^R \sin x \, \mathrm{d}x \\ &= \lim_{R \to +\infty} (-\cos x) \Big|_0^R \\ &= \lim_{R \to +\infty} (-\cos R - (-\cos 0))\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \; a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(f)

Izračunajte integral $\int_0^{\infty} \sin x \,\mathrm{d}x$.

Rješenje. Imamo

$$\begin{aligned}
\int_0^{\infty} \sin x \,\mathrm{d}x &= \lim_{R\to+\infty} \int_0^{R} \sin x \,\mathrm{d}x \\
&= \lim_{R\to+\infty} (-\cos x)\Big|_0^R \\
&= \lim_{R\to+\infty} (-\cos R - (-\cos 0)) \\
&= \lim_{R\to+\infty} (-\cos R + 1)
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x \,\mathrm{d}x = e^x + C$$
$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x \,\mathrm{d}x = \sin x + C$$
$$\int \sin x \,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 53(f)

Izračunajte integral $\int_0^{\infty} \sin x \, \mathrm{d}x$.

Rješenje. Imamo

$$\begin{aligned}
\int_0^{\infty} \sin x \, \mathrm{d}x &= \lim_{R \to +\infty} \int_0^{R} \sin x \, \mathrm{d}x \\
&= \lim_{R \to +\infty} (-\cos x)\Big|_0^R \\
&= \lim_{R \to +\infty} (-\cos R - (-\cos 0)) \\
&= \lim_{R \to +\infty} (-\cos R + 1) \ \text{ ne postoji.}
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x \, \mathrm{d}x = e^x + C$$
$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \ a \neq 1)$$
$$\int \cos x \, \mathrm{d}x = \sin x + C$$
$$\int \sin x \, \mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$